A comparison of reference conditions in floating frame formulations that use absolute interface coordinates

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EXTENDED ABSTRACT

The dynamics of flexible multibody systems composed of multiple flexible bodies which can undergo large rigid body rotations, is often studied by formulating the problem with floating frames of reference. In this formulation, a floating frame is defined for each body such that the overall rigid body motion of each body is described by the absolute coordinates of the floating frame. The elastic deformation is described locally with respect to the floating frame. Figure 1 shows this description schematically for an arbitrary body, where O denotes the global frame, j denotes the floating frame and i denotes a material point on the body. Under infinitesimal assumption, the local elastic deformation can be described by a set of orthogonal basis functions and generalized coordinates. The use of orthogonal functions for model reduction is well established and helps to reduce computational time. This makes the floating frame of reference description the preferred method for multibody dynamic problems over geometrically non-linear FEM and corotational frame formulations.

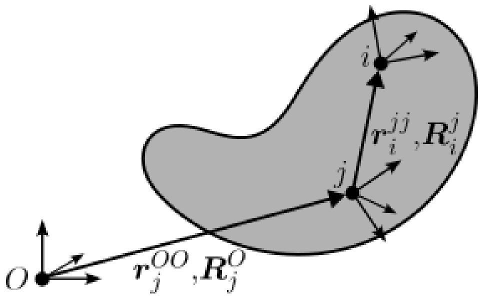

Figure 1: A schematic illustration of the coordinates used in the floating frame of reference formulation, with O denoting the global frame, j denoting the floating frame and i denoting a material point on the rigid body.

A disadvantage of the floating frame of reference formulation concerns the inclusion of the kinematic constraints, which are nonlinear equations in terms of the absolute floating frame coordinates and local generalized coordinates. Therefore, Lagrange multipliers are required to include the kinematic constraints in the equations of motion. The kinematic constraints can be strongly simplified if the equations of motion are expressed in the absolute interface or boundary coordinates (the coordinates of the nodes at which the bodies are connected to each other or to the real world). This is schematically illustrated in Figure 2, where B_1 and B_2 denote two boundary nodes. As a result, the constrained equations of motion would be of ordinary differential form, instead of being differential-algebraic.

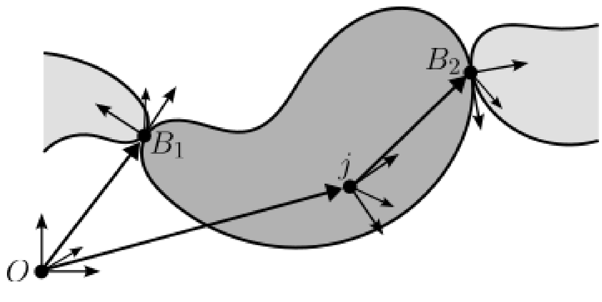

Figure 2: A schematic illustration of the boundary nodes described as absolute coordinates and floating frame of reference coordinates.

To achieve the transformation from absolute floating frame and local generalized coordinates to absolute boundary coordinates, a unique transformation matrix must be defined which requires reference conditions that remove the redundancy between the three coordinate sets. The redundancy is present, because the elastic deformation can contain rigid body motion depending on the definition of the floating frame and the choice of the mode shapes. One commonly used option for the mode shapes when using the absolute boundary coordinates are the Craig-Bampton modes, due to the simple relationship between the generalized coordinates and the boundary node coordinates. However, since linear combinations of the Craig-Bampton modes can describe rigid body motion, this choice of mode shapes does not eliminate the aforementioned redundancy. Therefore, the reference conditions must be six position-level constraints relating the absolute floating frame coordinates to the absolute boundary node coordinates.

Three well established reference conditions of this kind are described by Cardona [1,2], and Ellenbroek and Schilder [3]. In [1], Cardona defines a floating frame attached to one of the boundary nodes. To establish a floating frame which is closer to the center of mass and does not introduce a distinction between the boundary nodes, Cardona [2] defined the floating frame as a weighted average of the movement of the boundary nodes as well. Ellenbroek and Schilder [3] developed a formulation in which the floating frame is attached to the node at the center of mass, and the elastic deformation is restricted to be zero at this node.

Direct comparison between reference conditions is complicated by the formulations being derived independently, leaving the choice of reference conditions to the preference and experience of the user. In this work, it will be shown that for each method it is possible to derive a transformation matrix of the form

$$\begin{bmatrix} \delta \boldsymbol{q}_j^{OO} \\ \delta \boldsymbol{q}_B^{jj} \end{bmatrix} = \begin{bmatrix} \boldsymbol{T}_j \\ \boldsymbol{T}_B \end{bmatrix} \delta \boldsymbol{q}_B^{OO} = \boldsymbol{T} \delta \boldsymbol{q}_B^{OO} \tag{1}$$

with δ denoting the variation of the coordinates $\boldsymbol{q}_c^{ab}$ of frame c relative to frame b expressed in frame a. The transformation matrices $\boldsymbol{T}_j$ and $\boldsymbol{T}_B$ transform the absolute boundary coordinates to absolute floating frame coordinates and local generalized coordinates respectively. Using the transformation matrices, one can convert the original equations of motion of one body expressed in absolute floating frame coordinates and local generalized coordinates

$$\boldsymbol{M}^O \begin{bmatrix} \ddot{\boldsymbol{q}}_j^{OO} \\ \ddot{\boldsymbol{q}}_B^{jj} \end{bmatrix} + \boldsymbol{C}^O \begin{bmatrix} \dot{\boldsymbol{q}}_j^{OO} \\ \dot{\boldsymbol{q}}_B^{jj} \end{bmatrix} + \boldsymbol{K}^j \begin{bmatrix} \boldsymbol{q}_j^{OO} \\ \boldsymbol{q}_B^{jj} \end{bmatrix} = \boldsymbol{Q}^O \tag{2}$$

to the equations of motion in terms of absolute boundary coordinates in the form

$$\boldsymbol{T}^T \boldsymbol{M}^O \boldsymbol{T} \ddot{\boldsymbol{q}}_B^{OO} + \boldsymbol{T}^T \left(\boldsymbol{M}^O \dot{\boldsymbol{T}} + \boldsymbol{C}^O \boldsymbol{T} \right) \dot{\boldsymbol{q}}_B^{OO} + \boldsymbol{T}^T \boldsymbol{K}^j \boldsymbol{q}_B^{OO} = \boldsymbol{T}^T \boldsymbol{Q}^O \tag{3}$$

Here, $\boldsymbol{M}^O$ and $\boldsymbol{C}^O$ are the global mass and velocity dependent matrices, $\boldsymbol{K}^j$ is the local stiffness matrix, and $\boldsymbol{Q}^O$ is the global vector of applied forces. This allows the implementation of the different formulations while changing solely the reference condition. As such, comparing the different formulations for accuracy is relatively unambiguous. Furthermore, the transformation matrices for the different formulations can be compared with respect to computational efficiency. The comparison between the reference conditions will provide the reader with the means to make a substantiated choice between the developed formulations.

The derivation of the transformation matrices of the three described methods developed by Cardona, and Ellenbroek and Schilder will be elucidated in the presentation. Furthermore, similarities and differences in the shape of the transformation matrices will be highlighted, and the performance of the resulting formulations will be evaluated using a relevant benchmark problem.

References

[1] A. Cardona and M. Géradin. A superelement formulation for mechanism analysis. Computer Methods in Applied Mechanics and Engineering, 100:1-29, 1992.

[2] A. Cardona. Superelement modelling in flexible multibody dynamics. Multibody System Dynamics, 4:245-266, 2000.

[3] M. Ellenbroek and J. Schilder. On the use of absolute interface coordinates in the floating frame of reference formulation for flexible multibody dynamics. Multibody System Dynamics, 43:193-208, 2018.